light, at an hundred foot diſtance, and that at an hundred and twenty foot diſtance I could diſcern ſome of the words. When I made this tryal, its Aperture (defined next the Eye) was equivalent to more than an inch and a third part of the Object-metall. This may be of ſome uſe to thoſe that ſhall endeavour any thing in *Reflexions*; for hereby they will in ſome meaſure be enabled to judge of the goodneſs of their Inſtruments, &c.

N. B. The Reader may expect in the *next Month* another Letter, which came ſomewhat too late to be here inſerted; containing a *Table*, calculated by the ſame Mr. *Newton*, about the ſeveral *Apertures* and *Charges* anſwering the ſeveral *Lengths* of theſe Teleſcopes.

EPITOME

Binæ Methodi *Tangentium* Doctoris *Johannis Walliſii* Geom. Prof. Saviliani *Oxoniæ*; aliàs fuſius & explicatius ab ipſo traditæ, hîc verò ob anguſtiam loci compendifactæ: In quarum Schematiſmis ſi forſan literæ quædam redundaverint, illæ ad ea pertinere cenſendæ ſunt, quæ in ampliori ejuſdem Scripto continentur, hîc vero dictâ de cauſa omittantur.

Habes hic (Clariſſime vir) eorum ſummam (ſtrictim traditam) quæ fuſiùs ſcripſeram, meas de Tangentibus Methodos *ſpectantia; duas potiſſimum quibus præſertim utor; alteram in Speciebus, alteram in Lineis; utramque generali formâ facilè explicabilem.*

Priorem adhibeo Con.Sect.prop.23,30,36,46,49. *& paſſim alibi. Quæ hæc eſt.*

Expoſitâ Curvâ Aa,(putà Parabola, fig. 4.) *quam in a tangat aF, diametro VDA occurrens in F; ordinatim applicentur aV, & DOT curvæ in O & tangenti in T occurrens. Ponatur autem* $Va = \mathrm{b}$, $VA = \mathrm{v}$, $VF = \mathrm{f}$, $VD = \mathrm{a}$, *adeóque* $DA = \mathrm{v} \pm \mathrm{a}$, $DF = \mathrm{f} \pm \mathrm{a}$;

Eſt (propter ſimilia triangula) $VF.DF :: Va.\ DT = \frac{\mathrm{f} \pm \mathrm{a}}{\mathrm{f}}\mathrm{b}$.

Item, ſi tangens ſit ultra curvam, $DT \gtrdot DO$; *ſi citra,* $DT \lessdot DO$: *Nempe,* $DT = DO$ *ſi intelligatur D in V; ſed, ſi extra V, DT vel DO major prout tangens eſt ultra citrave curvam.*

Tum,

Tum, habita ipsius DO designatione quæ sit expositæ curvæ accommodata; (*puta*, *in Parabola*, *propter AV . AD : : Vaq .* $DOq = \frac{v \pm a}{v} b^2$; $DO = b\sqrt{\frac{v \pm a}{v}}$:) *fiat debita reductio*, (*puta*, *propter* $\frac{f \pm a}{f} b \gtreqless b\sqrt{\frac{v \pm a}{a}}$, *adeoque* $\frac{f^2 \pm 2fa + a^2}{f2} \gtreqless \frac{v \pm a}{v}$, & $f^2 v \pm 2fva + va^2 \gtreqless f^2 v \pm f^2 a$; *deletis utrinque æqualibus*, *hoc est*, *iis omnibus in quibus* a *non conspicitur*; *cæterisque per* $\pm$ a *divisis* : $2fv \pm va \gtreqless f^2$.)

Tandem (*qui methodi nucleus est*) *posito D in V*, (*quò sit* $a = o$, *adeoque evanescant ipsius multipla omnia*,) *æquatio exhibebit* f *quæsitam* (*puta* $2fv \pm va = 2fv = f^2$, *adeoque* $2v = f$.)

Hanc (*locis citatis*) *accommodatam videas Parabolæ*, *Ellipsi*, *Circulove*, *Hyperbolæ*, *Paraboloidibus omnibus*, (*quibus* & *harum Reciprocas accenseo*,) *atque alibi aliis*.

Cissoidi (fig.5.) *sic accommodes*. *Est* (*per* cap. 5. pr.29. de Motu) $Va = b = \frac{v^2}{s = \sqrt{vh}}$, (*posito* s *pro sinu recto in circulo generante*, *cujus radius* r, *sinus versus* v, & $2r - v = h$, & $h - v = 2x$,) *adeoq*; (*substitutis* $v \pm a$ *pro* v, & $h \mp a$ *pro* h,) $\frac{v^2 \pm 2va + a^2}{\sqrt{: vh \pm 2xa - a^2 :}}$ (= *D O*) $\gtreqless$ (*DT* $= \frac{f \pm a}{f} b =$) $\frac{f \pm a}{f} \times \frac{v^2}{\sqrt{vh}}$. *Ergo* (*sumptis quadratis*, & *multiplicando decussatim*,) $f^2 v^5 h + 6f^2 v^3 h a^2 + f^2 v h a^4 \pm 4f^2 v^4 h a \pm 4f^2 v^2 h a^3 \gtreqless f^2 v^5 h \pm 2fv^5 h a + v^5 h a^2 \pm 2f^2 v^4 x a + 4fv^4 x a^2 \pm 2v^4 x a^3 - f^2 v^4 a^2 \mp 2fv^4 a^3 - v^4 a^4$: *item* (*deletis utrinque æqualibus*, *cæterisque per* $\pm$ va *divisis*) $\pm 6f^2 v^2 h a \pm f^2 h a^3 + 4f^2 v^3 h + 4f^2 v h a^2 \gtreqless 2fv^4 h \pm v^4 h a + 2f^2 v^3 x \pm 4fv^3 x a + 2v^3 x a^2 \mp f^2 v^3 a - 2fv^3 a^2 \mp v^3 a^3$. *Denique* (*posito D in V*, *quo evanescat* a *cum suis multiplis*, *cæterisque per* 2 fv^3 *divisis*) *fiet æquatio* $2fh - fx = vh$, *adeoque* $\frac{vh}{2h - x = r + h = 3r - v} = f$.

Idem succedet, *sumptâ*, *pro VA*, *diametro YA*, (*cui tangens occurrat in* Φ) *aliâ VC*. *Item*, *si exponeretur curva quæ ordinatas non habeat*, *sed quæ his æquipolleant*; *ut sunt*, *in Spirali*, *crescentes radii*.

Sed & calculi magna pars præverti potest; *omissis ab initio* (*utpote pòst rejiciendis*) *terminis iis ubi habetur* a^2 *vel superior hujus potestas*; *item iis in quibus nec* a *conspicitur*, *nec sunt in* a *ducendi*, (*utpote æqualibus utrinque prodituris*.) *Exempli gratiâ*,

In

In Conchoide, (fig 6.) *cujus ordinata $VM\alpha$ constat ex sinu recto $VM = s = \sqrt{vh}$, & tangente $M\alpha = CH = \frac{s}{\kappa} r$, (si sit $CP = CA = r$, adeoque $CH = AS$;) saltem $= \frac{s}{\kappa}\rho$ (posito $CP = \varrho$:) adeoque $V\alpha = b$* $= s + \frac{s}{\kappa} r = \frac{\kappa + r}{\kappa} s = \frac{h}{\kappa} s$, *saltem* $\frac{\kappa+\rho}{\kappa} s = \frac{\nu}{\kappa} s = \frac{\nu}{\kappa} \sqrt{vh}$ (*posito* $\kappa + \varrho = \nu$.) *Ergo* $DT = \frac{f \pm a}{a} b = \frac{f \pm a}{f\kappa} \nu \sqrt{vh} \geqq DO = \frac{\nu \mp a}{\kappa \mp a} \sqrt{:vh \pm 2\kappa a -:}$ (*omitto* a^2, *quia post delendum, indeque oriunda, & sic semper:*) &, *sumptis quadratis*, $\frac{f^2 \pm 2fa +}{f^2\kappa^2} \nu^2 vh \geqq \frac{\nu^2 vh \pm 2\nu^2\kappa a \mp 2\nu vha \pm}{\kappa^2 \mp 2\kappa a -}$ (*hoc est*, $\overline{>}$ *supra, sed* $\overline{<}$ *infra, punctum flexus contrarii.*) *Et, decussatim multiplicando; omissis (ut præcipitur)* $f^2\kappa^2\nu^2 vh$ *utrobique, omnibusque* a^2 *multiplis; cæterisque per* $\pm a$ *divisis*; $2f\nu^2 vh\kappa^2 - 2f^2 \nu^2 vh\kappa \pm \geqq 2f^2\nu^2\kappa^3 - 2f^2\nu vh\kappa^2$: *adeoque* (*posito D in V*,) $\nu vh\kappa = f\nu vh + f\nu\kappa^2 - fvh\kappa = f\nu r^2 - fvh\kappa$ (*propter* $vh + \kappa^2 = s^2 + \kappa^2 = r^2$,) & $f = \frac{vh = s^2}{\nu r^2 - vh\kappa} \nu\kappa$. *Et quidem, in primaria*, (*propter* $h = \nu$,) $f = \frac{s^2\kappa}{r^2 - \kappa r^2}$

In Figura Tangentium (fig.7.) *quæ à Conchoide differt, exempto quadrante genitore; idem erit processus, nisi quod, propter* $V\alpha = M\alpha = \frac{\rho}{\kappa} s$ (*non* $\frac{\nu}{\kappa} s$,) *prodibit* (*sive in primaria, sive in protracta contractave*,) $f = \frac{vh = s^2}{r^2} \kappa$.

In Figura Secantium (fig.8.) *propter* $V\alpha = b = \frac{r^2}{\kappa}$; *erit* $DO = \frac{r^2}{\kappa \mp a} \overline{>} \frac{f \pm a}{f\kappa} r^2 = DT$. *adeoque* $f = \kappa$.

Cumque hæc curva sit Hyberbola (*per* pr.30.cap.5.& pr.1.cap. 15. de Motu,) *cujus Asymptotæ* CA, $C\beta$: *eadem tangens habetur per* pr. 36.Con. sect. *Cumque ordinatæ ad asymptotas* (*per* pr. 94, 95, Arith. Infin.) *fiat* series Reciproca Primanorum (*quæ ad Paraboloïdium genus spectat, verticem habens C, exponentem* -1,) *habetur eadem tangens per* prop. 49. Con. Sect. (*eademque est expedita methodus pro hyperbolæ cujusvis tangente per asymptotam inveniendâ.*) *Quippe, in Paraboloïdibus omnibus, ut intercepta diameter VC, ad VF, sic* 1 *ad exponentem: hoc est, in præsenti casu, ut* 1 *ad* -1; *adeoque* $VC = VF$, *sed* (*propter contraria signa* $+ -$) *ad contrarias partes.*

Notandum hic; in Parabolâ, Paraboloïde, Hyperbolâ, Ellipsi, &c. figurâve Sinuum (rectorum, versorumve,) Arcuum, Tangentium,

gentium, Secantium, &c. aliâve cujus constructio est Similaris; protractio contractiove figuræ (seu mutatio Lateris recti, aut quod hujus instar est,) non mutat punctum F, (eo quod Latus-rectum æquationem quæ longitudinem VF determinat non ingreditur, utut eam ingrediatur quæ determinat longitudinem Vα, mutetque angulos ad α & F:) sed ubi constructio est Dissimilaris, ut in Cycloïde & Conchoïde (propter ordinatam illic ex Sinu & Arcu, hîc ex Sinu & Tangente, conflatam,) aliisque istiusmodi, res secus est: eò quòd una pars (ut Arcus in Cycloïde & Tangens in Conchoïde) protrahitur contrahiturve, manente alterâ (putà, in utrisque Sinu recto) ut in primariâ.

Idemque dicendum de Angulo applicationis (ad V,) cujus mutatio non mutat longitudinem VF, sed neque Vα, quia neutrius ingreditur æquationem. Atque hinc fit, quòd in figura Scalena, quæ ordinatas contrarias, utrinque ad V positas, spectant tangentes, utut inæquales, in eodem F conveniant. Sed & (ut hoc obiter moneam) quadratorum aggregatum habent idem atque in erectâ; nempe semper $= 2\,V\alpha q + 2\,VFq$.

Estque hæc mihi methodus pro Maximis & Minimis *in omne genus quantitatibus.*

Methodus altera (secundùm tradita de Angulo Contactus *&* Arithm. Infin.) *curvam considerat tanquam ex particulis conflatam infinitè exiguis, sed certam positionem habentibus; eandem nempe (propter contactûs angulum sive nullius magnitudinis sive infinitè exiguæ) cum recta ibidem tangente: adeoque cum hac (respectu cujusvis rectæ) pariter declivem, (ut est Montis Aα* fig. 4, 5, *declivitas in α, eadem quæ tangentis α F.) Cujus ergo quæque particula (per* cap. 2. de Motu) *est in eâ ratione magis longa (quàm est respectiva expositæ rectæ particula æquè-alta) quâ est minùs declivis; puta αT quàm VD: Unde, propter mutatam in singulis punctis declivitatem, oritur series longitudinum inæqualium in curvâ, seriei æqualium in rectâ, respondens; curvæ ad rectam rationem exhibens. Atque hinc methodus mea pro* curvis rectificandis, (schol. prop. 38. Ar. Infin. *insinuata,) quam prosequor tractatu de* Ἐυθύνσει, *item* de motu cap. 5. prop. 13. & seqq. *Cujus aliqua pars est hæc de Tangentibus, ut quæ non totam declivitatum seriem perpendit, sed eam quæ est in exposito puncto.*

Hanc respectivam particularum longitudinem, aliàs insinuatum

eunt

eunt (motu forinsecus assumpto) per motuum quibus transfigantur ἰσοχρόνων *celeritatem. (Quippe idem est, in Motu, Celeritas, atque hæc, in Situ (propter positionem obliquam seu minus declivem) respectiva Longitudo.) Aptissimè quidem in lineis a motu primitus oriundis, (putà, Cycloide, Conchoide, Spirali, Quadratrice, &c.) nec ineptè in aliis, quæ fingi saltem possunt istiusmodi motibus describi.*

Præsumo autem (ex prop. 15. cap. 2. de Motu) *eam esse curvæ in quovis puncto directionem, adeoque & declivitatem, quæ est rectæ ibidem tangentis: Item (ex* prop. 6. cap. 10.) *Motûs compositi directionem esse in Diagonio parallelogrammi, cujus latera & anguli exhibeant componentium celeritates & directiones.*

Intelligatur jam (fig. 4.) *Aα* parabola, *describi motu composito, ex æquabili secundum AΥ vel Vα, cujus itaque particulæ* ἰσόχρονοι *(per* pr. 3. cap. 10. de Motu) *sunt series Primanorum, quæ ad seriem totidem ultimæ æqualium, (hoc est, ad rectam* ἰσόχρονον *celeritate in α acquisitâ transigendam,) est ut* 1 *ad* 2, (*per* Ar. Infin. pr. 64. *vel* pr. 1. cap. 5. de Motu.) *Adeoque, sumpta VF = 2 VA, & completo FVαν parallelogrammo; juncta αF est Tangens.*

Idem similiter obtinebitur in Paraboloidibus quibuscunque, ope prop. 2, 5, 6, 7, de Motu.

Atque inde facile (vel ex iisdem principiis) ostenditur; si intelligatur Fig. AΥυ sic constituta, ut momenta (respectu AF) ordinatorum Υυ, yω, sint ipsis Υα, yO, ordinatis proportionalia; erunt Celeritates acquisitæ in α, o, seu V, D, (positâ AΥ linea motûs æquabilis) rectis Υυ, yω, proportionales: Et consequenter, ut AυΥ (illarum aggregatum) ad AFυΥ (aggregatum totidem maximæ æqualium,) sic VA (aggregatum celeritatum seu particularum crescentium) ad (aggregatum totidem maximæ æqualium) ΥF.

Spiralis *ASα* (fig. 9.) *punctum α designatur motu composito ex recto per Aα, & circulari per Vα, æquabilibus utrisque &* ἰσοχρόνοις. *Ergo, sumpta circuli tangente ων=αV, & completo AωF parallelogrammo; juncta αF Spiralem tanget.*

Unde statim emergit Archimedea quadratura (sive Circuli sive Sectoris cujusvis) propter AF=ων=αV.

Sin motuum alter, puta Aα, sit acceleratus vel retardatus; pro

A,

αA, sumenda erit αB (in ea ad illum ratione quam illa postulat acceleratio seu retardatio,) eritque diagonium $\alpha\beta$, Tangens quæsita.

Quadratricis *$A\alpha B$* (fig. 10.) *punctum α designatur motu composito ex recto per $\upsilon\alpha$, & circulari in $\Upsilon\alpha$ (æquabilibus & ἰσοχρόνοις.) Ergo, sumpta tangente $\alpha V = \alpha\Upsilon$, & completo parallelogrammo $V\omega F$, juncta αF tanget Quadratricem.*

Atque hinc alia quadratura, per Tangentem quadratricis, propter $\upsilon F = \alpha V = \alpha\Upsilon$.

Illa per quadratricis Basin, sic elicitur. Positis $CA = r$, $AQ = q$, $\Upsilon Z = \chi$, $QR = a$. Erit (propter Quadratricis constructionem) $AQ.RQ :: AC.\alpha AE = .\frac{a}{q}r :: \Upsilon Z.\alpha Z = \frac{a}{q}\chi$. Estque $\alpha Z > \alpha E$ sumpto ubivis in AB puncto α, præterquam in B, quo casu (evanescente utraque) erit $\alpha Z = \alpha E$, adeoque $\chi = r$; hoc est, $\Upsilon Z = XB = AC$. Sed & υE communis tangens utrique curvæ XB, AB.

Cycloïdis (fig. 11.) *punctum α describitur motu composito, ex recto in αV, & circulari in $\alpha\beta$ (æquabilibus & æquè velocibus.) Ergo, sumpta tangente $\alpha\omega = \alpha V$, & completo $V\omega F$ parallelogrammo, juncta αF Cycloidem tanget. Et quidem, propter Ang. $\upsilon\alpha F$ ($= \alpha\beta F = \frac{1}{2}\alpha CF$) $= \frac{1}{2}\upsilon\alpha V$, occurret circuli $\alpha\beta$ erectæ diametro in vertice.*

In secundariis (contractâ protractâve) sumenda erit $\alpha\omega$ ad αV, in ea ratione major minorve, quâ est celeritas motûs circularis ad celeritatem recti.

In Figura Arcuum, Sinuumve, (fig. 12.) *procedendum ut in Cycloide, nisi quòd (propter exemptum semicirculum genitorem) pro tangente $\alpha\omega$ illic (quæ hîc est $\alpha\tau$) sumenda erit erecta $\alpha\omega$ æqualta.*

Conchoidis (fig. 6.) *punctum α designatur motu composito, ex æquabili circulari in $\alpha\beta$ (hujusve tangente $\alpha\omega$) & recto in $\alpha\Upsilon$ accelerato pro incremento tangentium: quæ quidem acceleratio duplex est, altera propter declivitatis angulum $\beta\alpha\Upsilon$, hoc est, $\upsilon\alpha\Upsilon$, continuè crescentem; altera propter radii in secantem protractionem, continuè item crescentem. Propter priorem, ducta tangente $\alpha\upsilon$ (quæ occurrat in υ regulæ CH,) recta $\upsilon\zeta$ (parallela rectæ $PH\alpha$,) occurrat $\alpha\Upsilon$ in ζ: Propter posteriorem; eadem $\upsilon\zeta$ protracta occurrat tangenti verticis in Z: indeque $Z\Upsilon$ rectæ $\upsilon\alpha X$ parallela; adeoque $\alpha\Upsilon = XZ$.*

Mmmm $\alpha\zeta$

$\alpha\zeta$ :: CM . μS :: $P\mu$. PH. *Completo denique* $\Upsilon\alpha\upsilon f$ *parallelogrammo, juncta* αf *tanget conchoidem.*

In secundariis (ubi non est $CP=CA$,*) sumenda erit* $\alpha\Upsilon$ *ad jam designatam, ut est* CP *ad* CA.

In Figura Tangentium (fig. 7.) *propter exemptum Conchoidi quadrantem genitorem, pro tangente* $\alpha\upsilon$ *illic (quæ hîc est* $\alpha\tau$*) sumenda erit erecta* $\alpha\upsilon$ *æquè-alta.*

Pluribus exemplis proferendis supersedeo. Moneo tamen, utramvis Methodum, utut tangentibus rectis hîc accommodatam, extendi posse ad mutuum Curvarum tactum. Puta; si, pro $FV\alpha$ *triangulo* (fig. 4, 5,) *intelligatur Hyperbola; recta DT, quæ hîc insignitur charactere qui triangulo conveniat, subire tum debebit characterem Hyperboles; cujus vertex F simili processu quæratur. Similisque in posteriori methodo accommodatus est linearum ductus. Et quidem, cum curvam* $A\alpha$ *tangens recta* αF, *sit etiam tangens communis curvarum omnium, expositam ibidem tangentium; prout hîc, ex data* $A\alpha$ *curva quæritur recta* αF, *sic ex hac datâ (per eandem methodum inversam) quærenda erit alia tangens curva, modò satis sit determinata.*

Sed ampliandum non est. Tu itaque Vale.

Tuus

Oxoniæ *die* 15.
Febr. 1671.

Johannes Wallis.

Extract

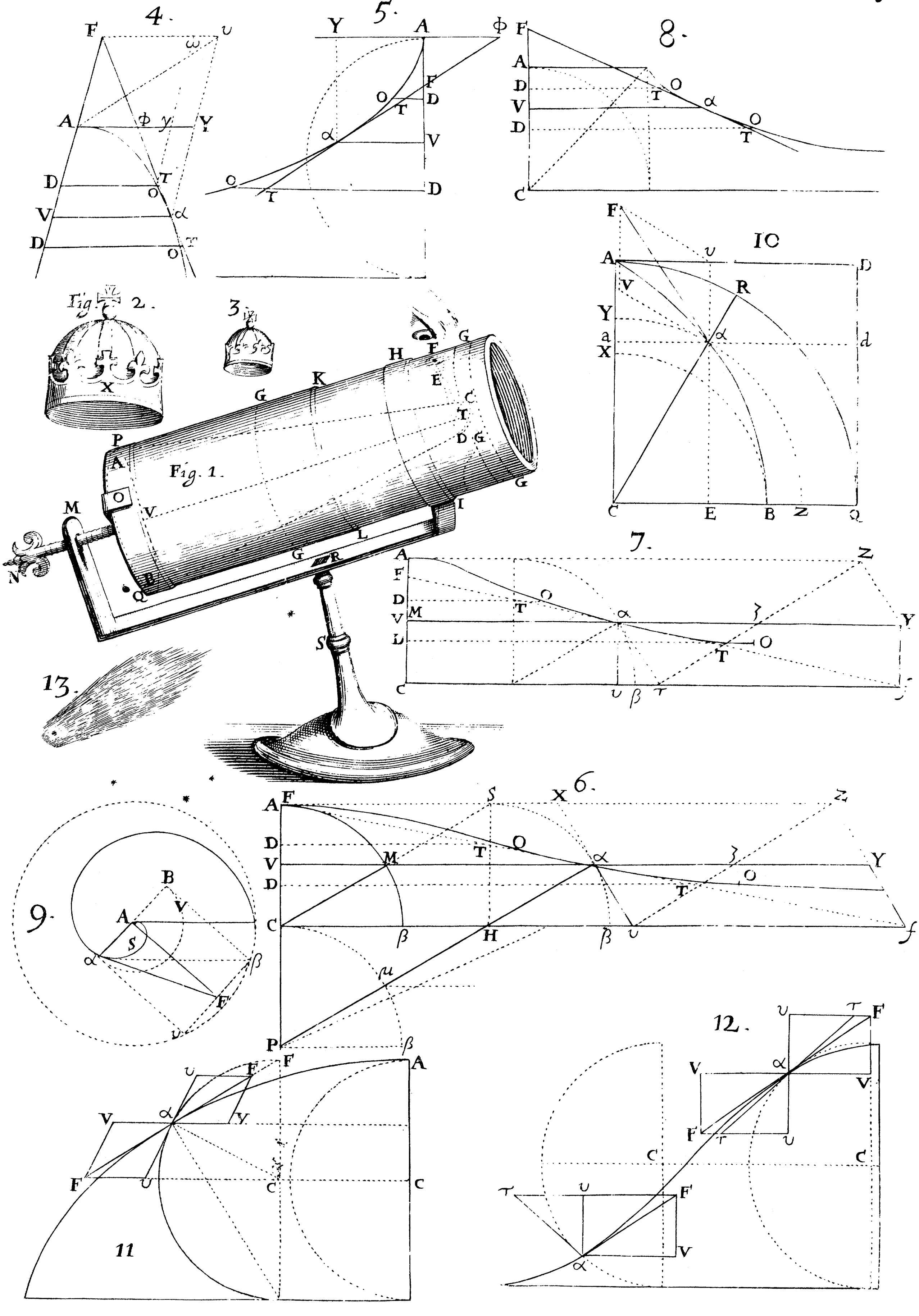
Fig. 1.
Fig. 2.
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